MAS220 ALGEBRA, BASIC REVISION GUIDE

Ideally you will read through the notes completely, following up all the internal references, studying all the examples that illustrate the meaning or significance of the definitions and propositions. At this stage just forget about exams and questions, rather focussing on following what's going on and seeing the complete picture as you read through the whole thing. Then you might start work on either the sample exam or one of the two past papers, depending on what you've already spoiled and what you want to save for later. See what you can do without the solutions first. Then see how much more you can do with the help of the notes to look things up. Finally consult the annotated solutions, observing the various tips on how to approach problems. This year's exam will by similar in style to previous years', except it will begin with 17 marks (out of 60) on very short questions, many with Yes or No answers, like the tests.

Examples in lectures and the notes are precisely that. They are not intended to be model answers to questions. But every time I explain the thought process in coming up with a proof that I am going through, you can view that as advice on how to answer questions. Similarly the supplementary videos on homework contain much general advice on how to approach questions.

You will need to know key concepts, definitions and propositions/theorems, and be able to apply them or produce simple examples in such a way as to demonstrate understanding. You will not be asked to produce any proof or example that in principle you couldn't be expected to do as an unseen exercise. But familiarity with some of the arguments in proofs in the notes, or examples in the notes, or ideas used in the solutions of homework and tutorial problems, may increase the chances of you having the right ideas. The more you know and understand, the more you are likely to be able to do on the exam. In that sense it is open-ended and there is no list of examinable proofs, examples or definitions.

Some of you may find that a bit daunting, and may prefer a more modest goal to get you started on some productive revision. So I suggest looking at the following as a basic revision plan (omitting proofs unless explicitly stated otherwise). If you know this much thoroughly, I think you may find you do well on past papers.

1. Semester 1

(1) Definition 1.1.1, of a group.
(2) Definition 1.1.2, of an abelian group.
(3) Examples 1.1.4, 1.1.5, 1.1.6, 1.1.8.
(4) Examples 1.1.9 and 1.1.10 (S_n and O_2), including why (at least for $n \geq 3$ in the case of S_n) they are not abelian.
(5) Definition 1.2.1 (subgroup) and the Subgroup Criterion.
(6) Definition 1.5.9 (normal subgroup). Examples 1.5.3, 1.5.4, 1.5.7, 1.5.8.
(7) Middle of p.14, definitions of conjugacy class and centraliser, the relation between their orders and that of the group (when it is finite), and the two remarks immediately following.
(8) Bottom of p.14, Theorem 1.6.4 and following example. Understanding of how conjugation and conjugacy classes work in S_n.
(9) Proposition 1.6.6, Examples 1.6.7, 1.6.8.
(10) Definition 1.8.1, group homomorphism.
(11) Definition 1.8.10, the kernel of a homomorphism.
(12) Proposition 1.8.11 (the kernel is a normal subgroup) and its proof.
(13) Theorem 1.8.18, First Isomorphism Theorem for group homomorphisms. Illustration by Examples 1.8.21, 1.8.22, 1.8.23.

(14) Definitions 2.1.1, 2.1.3 and 2.1.5 (ring, commutative ring and field), illustrated by Examples 2.1.2, 2.1.4 and 2.1.6.
(15) Proposition 2.2.1 and its proof.
(16) Definition 2.4.1 (subring) and the Subring Criterion. Example 2.4.3 (Gaussian integers).
(17) Section 2.6. Definition of matrix ring $M_n(R)$, and its non-commutativity when $n \geq 2$.
(18) Definition 2.9.1 (ring homomorphism) and illustration by Examples 2.9.3, 2.9.4, 2.9.9.
(19) Definition 3.1.1 (integral domain). Proposition 3.1.2, Example 3.1.3.
(20) Definition 3.2.1 (Euclidean domain), Examples 3.2.2, 3.2.3, 3.2.5 (statement).
(21) Definitions 2.8.1, 3.1.5, 3.1.7, 3.1.9, Examples 3.1.10, 3.1.11, 3.1.12.
(22) Theorem 3.3.2, Example 3.3.3, Theorem 3.5.2.
(23) Theorem 3.6.8, Examples 3.6.9, 3.6.10, 3.6.11, 3.6.12.
(24) Proposition 3.8.1, Theorem 3.8.2.

2. Semester 2

(1) Definition 1.4.1 (subspace) and Subspace Criterion.
(2) Definition 1.4.2 (Span). Theorem 1.4.3. Example 1.4.6, Definitions 1.4.7, 1.4.8.
(3) Definition 1.6.1 (null space). Theorem 1.6.2 and its proof.
(4) Section 1.6. Ability to solve a homogeneous system of linear equations (i.e. to find the null space of a matrix) by row-reduction to reduced row echelon form.
(5) Definitions 1.7.3, 1.7.6, 1.7.7 (linear dependence and independence, basis).
(6) Proposition 1.7.14.
(7) Definition 1.8.1 (linear map).
(8) Theorem 1.8.3 and its proof, Examples 1.8.6, 1.8.7.
(9) Definition 1.10.3 (dimension), and examples immediately following it.
(10) Definitions 1.11.2 (nullity), 1.11.7 (rank) and Theorem 1.11.8 (rank-nullity). Example 1.11.9.
(11) Example 1.12.8.
(12) Example 1.13.2.
(13) Definition 1.13.6 (eigenvector and eigenvalue).
(14) Theorem 1.13.8, Example 1.13.9.
(15) p.27, definition of the kernel of a linear map, and proof that it is a subspace (Proposition 1.14.1).
(16) Example 1.14.2 (null space as a kernel).
(17) Proposition 1.14.6 and its proof. Example 1.14.7.
(18) Theorem 1.14.10, Examples 1.14.11,1.14.12,1.14.13.
(19) Middle of p.30, three lines beginning "Concretely, if". Proposition 1.15.1, Example 1.15.3.
(20) Definition 2.1.1 (inner product space)
(21) Definitions 2.1.2, 2.1.3 (norm and orthogonality). Corollary 2.1.6 (angle).
(22) Definitions 2.2.1, 2.2.6 (orthogonal and orthonormal sets).
(23) Example 2.2.13 (Gram-Schmidt process).
(24) Definition 2.2.14 (orthogonal complement).
(25) Definitions 2.4.1 and 2.4.3 (adjoint and self-adjoint).
(26) Proposition 2.4.2 and its proof.
(27) Theorem 2.4.7 and its proof.